

03 Divisibility A

TAX IDENTIFICATION NUMBER

The tax identification number is an identifying number used for tax purposes. NIF is the Spanish name; the meaning is 'Número de Identificación Fiscal'.

It is the result to add a letter to the identity card number.

What is the purpose of this letter? It is a new control code to avoid mistakes in the process of reading this identification. The same aim that the control code in a bar code.

Logically this letter has a formula.

The letter is calculated from the resulting remainder of division of identity card number into 23 according to the following table:

0 T	1 R	2 W	3 A
4 G	5 M	6 Y	7 F
8 P	9 D	10 X	11 B
12 N	13 J	14 Z	15 S
16 Q	17 V	18 H	19 L
20 C	21 K	22 E	

Now if we introduce our NIF in a computer, the computer evaluates the truth for the letter according to the previous formula. If the result is different then the computer asks the number again. It is a way to check that we write the correct number for our identity card.

WORK IN PAIRS

One of you has a complete NIF and the other without the letter; and the other partner the opposite.

The objective of this activity is to guess the letter that you need and ask your partner if it is correct.

A



03 Divisibility _B

TIN (TAX IDENTIFICATION NUMBER)

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B



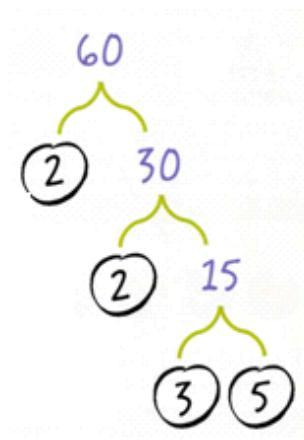
FACTOR TREES

You can make a diagram called a factor tree to find out the prime factors of a number.

To draw a factor tree for a number, take a pair of its factors and split each factor up into its own factor pairs, as shown below.

Keep splitting the factors until you reach a factor that you can't divide any more, then draw a circle around it.

These circled figures are the number's prime factors.



Your tree

Do a factor tree for the numbers: 150 and 2520.

And finally factorize the numbers.

1. MULTIPLES OF A NUMBER

Multiples of a number are the numbers we get multiplying the number by any natural number.

Multiples of 2: 2, 4, 6, 8,...

Multiples of 3: 3, 6, 9, 12,...

The multiples of a number are infinite.

Factor is the name of a number that is multiplying.

For example: $3 \cdot 5 \cdot 7$ has three factors.

2. DIVISORS OF A NUMBER

A number is divisor of another if the division remainder is zero.

For example, 5 is a divisor of 15.

If a number 'a' is divisor of another 'b' then 'b' is multiple of 'a'.

It is similar to the relation between a father and his son.

If **a** is father of **b** then **b** is son of **a**.

For example, 5 is divisor of 15 so 15 is multiple of 5.

DIVISIBILITY RULES

There are some rules about divisibility. We can find out some divisors by these rules. It is a quick way to work out some divisors of a number.

The most common and easy rules are for the numbers 2, 3, 5 and 11.

Divisible by:	If:	Examples (Y –yes–; N –no–)
2	The last digit is even (0,2,4,6,8)	12 8 Y; 12 9 N
3	The sum of the digits is divisible by 3	381 (3+8+1=12) 217 (2+1+7=10)
4	The last 2 digits are divisible by 4	13 12 70 19
5	The last digit is 0 or 5	17 5 80 9
6	The number is divisible by both 2 and 3	114 308
9	The sum of the digits is divisible by 9	1629 2013
10	The number ends in 0	22 0 22 1
11	The sum of the even digits minus the sum of the odd digits is either 0 or divisible by 11	1 3 64 ((3+4) - (1+6) = 0) Yes 372 9 2517 6

3. PRIME NUMBERS AND COMPOSITE NUMBERS

A prime number is the number that has only two divisors: 1 and itself.

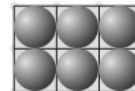
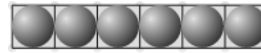
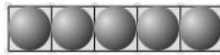
For example, 2, 3, 5 are prime numbers.

A composite number is the number that has some different divisor than 1 or itself.

For example, 18 is a composite number because it is divisible by 2. That is, $18 = 2 \cdot 9$

The prime numbers are called linear numbers because you can make only segment with them. The composite numbers are called rectangular numbers because you can make rectangles with them.

5 is prime number so it is linear number 6 is composite number so it is rectangular number



4. PRIME FACTORIZATION

The prime factorization of a number is the result to put the number as the product of the entire prime factors for the number.

For example, $30 = 2 \cdot 15 = 2 \cdot 3 \cdot 5$

30		2		30		2
10		15		3		15
0		0		5		5
				0		1

NUMBER OF DIVISORS OF A NUMBER

It is obtained by adding one unit to each exponent and multiplying the obtained results.

Example:

What is the number of different divisors for 60?

5. LEAST COMMON MULTIPLE

Common multiple of several numbers is a number that is multiple of all of them.

For example, 30 is common multiple of 2, 3 and 5.

The simplest way to get a common multiple is multiplying all the numbers.

For example, a common multiple of 5 and 9 is $5 \cdot 9 = 45$.

LCM is the abbreviation of least common multiple.

The LCM is the result of multiplying the entire prime factors with the highest exponent.

Example:

Work out the LCM of 45 and 60 by writing the first multiples for each and finding out the first common multiple in the list.

Work out the same by using the prime factorization.

6. HIGHEST COMMON FACTOR

Common divisor of several numbers is a number that divide all of them.

For example, 2 is a common divisor of 4, 10 and 12.

HCF is another name that is highest common factor. That is, the highest common divisor of all of them.

The HCF is the result of multiplying the entire prime common factors with the smallest exponent.

Example:

Work out the HCF of 45 and 60 by writing the first multiples for each and finding out the first common multiple in the list.

Work out the same by using the prime factorization.

Property

Relationship between the HCF and LCM of a and b : $\boxed{\text{HCF} \cdot \text{LCM} = a \cdot b}$

Example:

Could you check the property in the previous example.

EXERCISES AND PROBLEMS

1. **MULTIPLES OF A NUMBER**

1. Write three common multiples to 2, 3 and 5.
2. Write three common multiples to 7 and 9.

2. **DIVISORS OF A NUMBER**

3. Calculate all the divisors of the following numbers:
a. 105
b. 220
4. Calculate all divisors of 150.
5. Are the following numbers divisible by 2, 3, 4, 5, 6, 7, 9, 10 and 11? Answer without doing the division.

	2	3	4	5	7	6	9	10	11
924									
1287									
5550									
8725									

6. Write whether the following numbers are divisible by the indicated numbers by putting yes or no in each box. Do it by applying the divisibility rules.

	2	3	4	5	7	6	9	10	11
1368									
42720									
6000									

7. Dar un valor en cada caso de C que haga al número 81C divisible:
a. Entre 2 y 3.
b. Entre 3 y 5.
c. Entre 2, 3 y 5.
8. Dar un valor en cada caso de C que haga al número 75C divisible:
a. Entre 2 y 3.
b. Entre 3 y 5.
c. Entre 2, 3 y 5.
9. Change each letter by a digit to get a number divisible by the indicated number:

45a8	3
99b	2
21c40	6
4d40	3
ff2	6
1235g	6

3. **PRIME NUMBERS AND COMPOSITE NUMBERS**

10. Sieve of Eratosthenes.
Here are the first 100 natural numbers.

Cross out the composite numbers and made a circle around those that are prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. **PRIME FACTORIZATION**

11. Find the prime factorization of 540.
12. Find the prime factorization of 450.
13. Calculate the GCF and lcm of the following numbers:
 $a = 2^3 \cdot 3^2 \cdot 7^2 \cdot 11$; $b = 2^2 \cdot 3^4 \cdot 5^2 \cdot 7$; $c = 2^3 \cdot 3^3 \cdot 5^2 \cdot 13$
14. What is the number that has $2^3 \cdot 3^2$ as prime factorization?
15. What is the number whose prime factorization is $2^2 \cdot 3^4 \cdot 5^2$?

5. **LEAST COMMON MULTIPLE AND HIGHEST COMMON FACTOR**

16. Calculate the least common multiple and the highest common factor of 60 and 140.
17. Calculate the least common multiple and the highest common factor of 84 and 90.
18. Calculate the least common multiple and the highest common factor of 126 and 60.
19. Work out the lcm and the HCF of 72, 108 and 540.
20. Work out the lcm and the GCD of 140 y 168.
21. Work out the lcm and the HCF of 98, 140 and 350.
22. Calculate the least common multiple and the highest common factor of 54 and 126.
23. In a public garden the gardener water the grass every 12 days, the woodcutter cuts the branches and weeds every 15 days and the biologist fumigates every 18 days. They met in the garden the 1st January, how long does it take them to meet again?
24. Write the highest prime number that has two digits and explain why.
25. Calculate the letter that corresponds to the following identity card: 78659342.

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